Game-Theoretic Analysis of Selfish Secondary Users in Cognitive Radio Networks

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Abstract: **In this paper, we study the problem of selfish behavior of secondary users (SUs) based on cognitive radio (CR) with the presence of primary users (PUs). SUs are assumed to contend on a channel using the carrier sense multiple access with collision avoidance (CSMA/CA) and PUs do not consider transmission of SUs, where CSMA/CA protocols rely on the random deference of packets. SUs are vulnerable to selfish attacks by which selfish users could pick short random deference to obtain a larger share of the available bandwidth at the expense of other SUs. In this paper, game theory is used to study the systematic cheating of SUs in the presence of PUs in multichannel CR networks. We study two cases: A single cheater and multiple cheaters acting without any restraint. We identify the Pareto-optimal point of operation of a network with multiple cheaters and also derive the Nash equilibrium of the network. We use cooperative game theory to drive the Pareto optimality of selfish SUs without interfering with the activity of PUs. We show the influence of the activity of PUs in the equilibrium of the whole network.**

Index Terms: **Cognitive radio network, game theory, Nash equilibrium, Pareto optimality.**

I. INTRODUCTION

WIRELESS technology relies on frequency spectrum as a fundamental resource. While frequency allocation charts reveal that almost all frequency channels have already been assigned, traditional static spectrum allocation strategies cause temporal and geographical holes [1] of the spectrum usage in licensed channels. Cognitive radio (CR) is viewed as a novel approach for improving the utilization of this precious natural resource, the radio electromagnetic spectrum [2]. CR systems have two types of users: Primary users (PUs) and secondary users (SUs). PUs are not aware of the SUs's behavior and PUs do not need any specific functionality to coexist with SUs who are typically not licensed and responsible for avoiding interference with PUs' transmissions.

Medium access control (MAC) of a CR system is usually analyzed based on the Markov chain of carrier sense multiple access with collision avoidance (CSMA/CA) [3]. In this paper, we also adopt a Markov model and embed channelization into CSMA/CA by which SUs operate on multiple channels with low priority to provide strict quality of service (QoS) guarantee for PUs. Since the SUs use CSMA/CA, their transmission relies on random deference of packets for efficient use of the spectrum hole. Although it is assumed that all the SUs respect the rules of the protocol, there is a possibility of selfish SUs due to the programmable nature of the MAC. While selfish SUs avoid interference with PUs, they may access the spectrum hole aggressively.

We use game theory, which is a very powerful tool to study the selfish behavior of players. We elaborate on the behavior of such selfish SUs and its effect on the system by using the game theory. We assume the secondary users as players, the throughput they get as their payoff, and the size of the contention window as their move. We obtain the Pareto-optimal and the Nash equilibrium point of operation of such a system.

We organize our paper as follows. Section II addresses related work. In Section III, we describe our system model. In Section IV, we derive the throughput of SUs in the presence of PUs. In Section V, we present a game theoretic model of the system, and in Section VI we show numerical results. Finally, Section VII concludes the paper.

Manuscript received March 18, 2015 approved for publication by Saewoong Bahk, Co-Editor-in-Chief, March 20, 2015.

This research was supported by the Ministry of Science, ICT and Future Planning (MSIP), Korea, under the Global IT Talent support program (IITP-2015-R0618151003000 1002) supervised by the Institute for Information and Communication Technology Promotion (IITP).

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Digital Object Identifier 10.1109/JCN.2015.000074

II. RELATED WORK

The study of CSMA/CA deference mechanism through game theory models has been found in literatures. The earliest work is found in [4]. However, this work does not consider CR environment. Hence, the rest of this section focuses on CSMA/CA for CR systems. In [5], the authors propose a MAC scheme that embeds physical channels in a multichannel CSMA/CA network to provide strict QoS to PUs. For PUs, physical channels are provided using channelization method to ensure QoS whereas SUs are assumed to follow the CSMA/CA protocol. This scheme does not consider that any SU can cheat and get more throughput at the expense of other SUs. This problem will be addressed in our paper.

In [6], the authors study selfish behavior in CSMA/CA networks using the game theory and propose a distributed protocol to guide multiple selfish nodes to Pareto-optimal Nash equilibrium. The authors computed the Pareto-optimal point of operation of such a system, and study the equilibrium of dynamic games. Besides that, this work also proposed detection and a punishment technique against cheaters.

In [7], a predictable random backoff (PRB) algorithm is proposed to mitigate the impacts of selfish nodes on the network performance and in particular on well-behaved nodes. This algorithm is based on minor modifications of the IEEE 802.11 binary exponential backoff (BEB) and forces each node to generate a predictable backoff interval. Nodes that do not follow

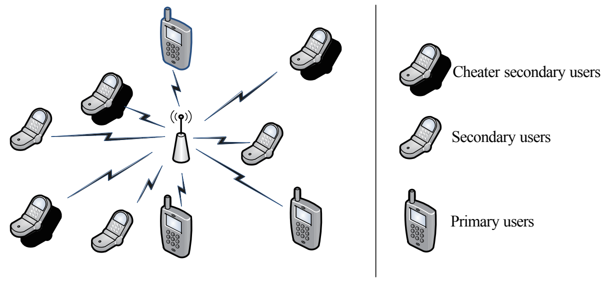

Fig. 1. Network setup of the proposed model.

the operation of PRB are easily detected and isolated. In [8], a game model is also proposed to interpret the IEEE 802.11 distributed coordination function mechanism. They design a simple Nash equilibrium backoff strategy to present a fairness game model. In [8], a game-theoretic approach is used to study selfish MAC-layer misbehavior under CSMA/CA, where the obtained bandwidth shares are considered as payoffs in a non-cooperative CSMA/CA game.

All the above references study the behavior of selfish users in CSMA/CA protocol. They all assume that the entire nodes have the same quality of service requirement but do not consider the presence of PUs. In our paper, we address the influence of PUs in addition to cheating behavior of SUs. We first derive the throughput of SUs in the presence of PUs and then investigate the cheating behavior of SUs.

Table 1. Summary of notations and symbols.

Notation	Representation of the symbol or symbol
P_p	The probability that each channel is occupied by PU
P_b	The blocking probability of channels due PUs activities
$N_{\rm CH}$	Number of channels
N_p	Number of PUs
N_s	Number of SUs
(i,j)	State where i represents backoff stage and j represents backoff counter
W	Contention window size
W_j	Contention window size at the backoff stage j
$CW_{\max}$	Maximum Contention window
$CW_{\min}$	Minimum contention window
$S(t)$	Stochastic backoff process time t
$b(t)$	stochastic process representing back of stage
p	Collision probabilty
$N_s^{\rm 1ch}$	Number of SUs in a single channel
τ	Access probability of each node
$b_{i,j}$	The stationary probability of state (i,j)
P^s	The probability successful transmission
$P^{\rm id}$	The probability channel idle
P^c	The probability channel collision
S	Throughput per channel
L	Packet length
T^s	The average time needed to transmit packet length L
$T^{\rm id}$	Duration of idle
T^c	Time spent in collision
N	Number of secondary Users
I	Numbers of cheaters
$U_i(s)$	Pay of function player i for strategy s
τ_i^c	Cheaters access probabilty
τ_i^w	Well behaved access probabilty
r_i	Throughput for player i
M	Maximum throughput that can reached by the cheaters
λ,α,β	Lagrangian multipliers

III. SYSTEM MODEL

Our network model consists of an access point and two tiers of users: The PU tier and SU tier as shown in Fig. 1. SUs are classified into well-behaved and cheating users. We assume that PUs have guaranteed QoS as explained in [5] and each user identifies its user type (primary or secondary) using the reserved field of association frames. We also assume that there is no hidden terminal problem. SUs use CSMA/CA based protocol to resolve the contention at MAC layer whereas channels assigned to PUs are orthogonal in multiple frequency channels. We consider orthogonal frequency hopping sequences in multiple frequency channels, e.g., Latin square, which exploits frequency diversity to avoid channel fading and interference. This model operates on multiple channels and SUs can randomly select an operating channel after sensing all channels.

The throughput of PUs is obtained under the assumption of no sensing error. Once they are admitted into the network, they just transmit their frames in the channels assigned by the AP. The Latin square hopping sequence is used to generate the probability of a PU for each secondary frame interval uniformly without a priori knowledge of hopping pattern at any SU [5]. Suppose the probability of each frequency channel being occupied by the transmission of a PU, P_p, is independent and identical of one another. It is a measure of the primary activity and is given by

$$P_p = \frac{N_p}{N_{\rm CH}}, \quad (1)$$

where N_p is the number of PUs and $N_{\rm CH}$ is the number of channels under the assumption that $N_p \leq N_{\rm CH}$. The probability that all channels are busy due to PUs depends on P_p and is given by

$$P_b = P_p^{N_{\rm CH}}. \quad (2)$$

IV. THROUGHPUT OF SECONDARY USERS IN THE PRESENCE OF PRIMARY USERS

In this section we determine the throughput of SUs in the presence of PUs. The throughput obtained in this section will be used to comupte the the utilization function in Section V. We modify the CSMA/CA analysis for the SUs is modified from [3]. For a given node (i.e., SU), each state is represented as (i,j) where i is the backoff stage and j is the current backoff counter. Let $b(t)$ be the stochastic process representing the backoff time counter for a given node. It has minimum $CW_{\min} = W$ and maximum $CW_{\max} = 2^m W$ where m represents the maximum backoff stage and W represents a contention window. The stochastic backoff process representing the backoff stage $(0,\cdots,m)$ of the SU at time t is given by $s(t)$. At each transmission attempt, every packet collides with a constant and independent probability of p regardless of the number of retransmissions.

The bi-dimensional process $\{s(t),b(t)\}$ is modeled with discrete-time Markov chain. It represents the operation of SUs in one channel among multi-channels in the presence of PUs; therefore we need to derive $N_s^{\rm 1ch}$, the number of SUs in a single channel statistically. The average number of SUs in one avail-

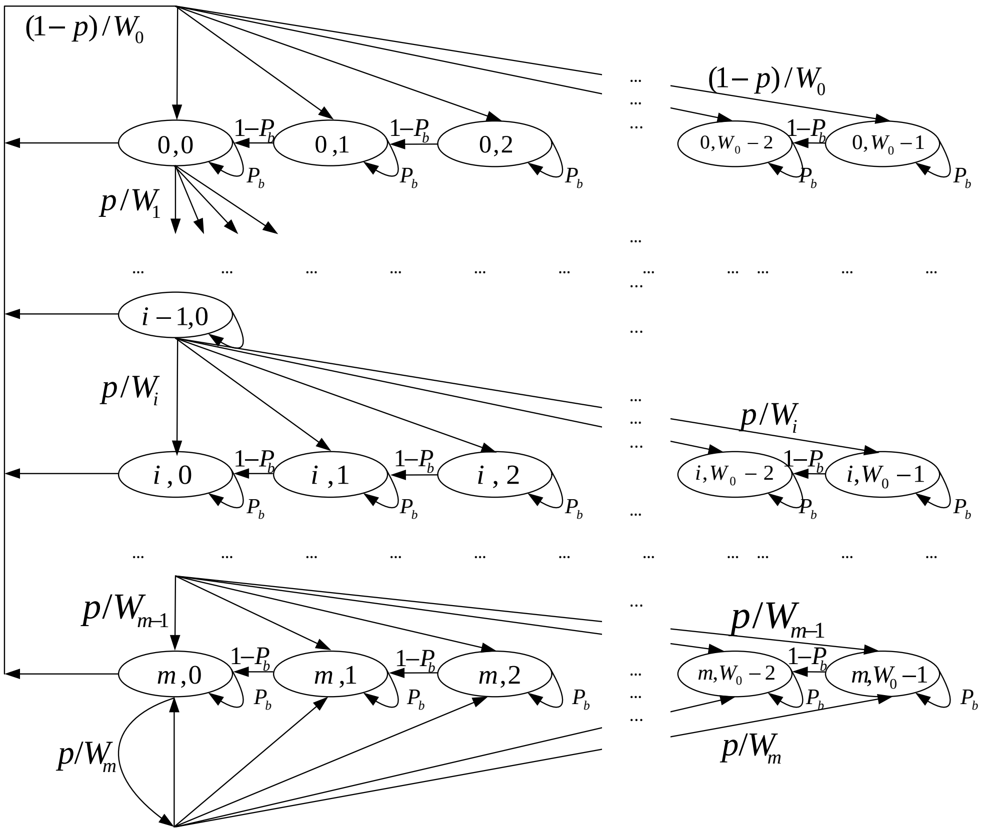

Fig. 2. Markov chain model for SUs based on CSMA/CA.

able channel is given by

$$P_b = P_p^{N_{\mathrm{CH}}}, \tag{3}$$

$$N_s^{1\mathrm{ch}} = \frac{N_s}{N_{\mathrm{CH}}(1-P_p)}, \tag{4}$$

where $N_{\mathrm{CH}}(1-P_p)$ is the average number of available free channels. The state transition probability for the Markov chain in Fig. 2 is given by

$$\begin{aligned} P\{j,k|j,k+1\} &= 1-P_b, & k \in (0,W_j-2), j \in (0,m)\\ P\{j,k|j,k\} &= P_b, & k \in (0,W_j-1), j \in (0,m)\\ P\{0,k|j,0\} &= \frac{p}{W_j}, & k \in (0,W_j-1), j \in (0,m)\\ P\{m,k|m,0\} &= \frac{p}{W_m}, & k \in (0,W_m-1), \end{aligned} \tag{5}$$

where W_j represent the contention in the jth stage. The first equation in (5) accounts for the fact that, when a SU has a positive backoff counter value of $k+1$, he decreases his backoff counter value if the channel is not occupied by a PU with probability $1-P_b$. If the channel is occupied by a PU with probability P_b, the SU maintains his backoff counter value as the same value k as shown by the second equation. The third equation accounts for the fact that a new packet following a successful packet transmission starts with backoff stage 0, and thus the backoff is initially uniformly chosen in the range $(0, W_0-1)$. The last two equations model the system after an unsuccessful transmission.

Let the stationary probability of state (i,j) be denoted by $b_{i,j}$. A transmission occurs when the backoff time counter is equal to zero. Thus, we can write the probability that a node transmits in a randomly chosen slot time as:

$$\tau = \sum_{i=1}^{M-1} b_{i,j}. \tag{6}$$

For the above Markov chain, the closed-form solution for $b_{i,0}$ as a function of p is obtained as follows. First, we can write the stationary distribution of the chain for $b_{i,0}$, $b_{m,o}$, and $b_{i,k}$:

$$\begin{cases} b_{i,0} &= \frac{p^i}{1-P_b} b_{0,0}, \qquad 0<i<m;\\ b_{m,0} &= \frac{p^i b_{0,0}}{(1-P_b)(1-p)},\\ b_{m,0} &= \frac{W_i-k}{W_i(1-P_b)} p b_{i-1,0}, \quad 0 \leqslant k \leqslant W_{i-1}. \end{cases} \tag{7}$$

The first and second expressions in (7) come from the fact that $b_{i-1}p = \frac{b_{i,0}}{1-P_b}$ for $0<i<m$ and $b_{m,0} = \frac{p}{1-p} b_{0,0}$ where $b_{m-1}p = \frac{b_{i,0}}{1-P_b}$. The third equation can be obtained by considering that $\sum_{i=0}^{m} b_{i,0} = \frac{b_{0,0}}{1-p}$ and taking the chain regularities into account (for $k \in (1, CW_i-1)$). We have

$$b_{i,k} = \frac{CW_i-k}{CW_i(1-P_b)} = \begin{cases} (1-p) = \sum_{j=0}^{m} b_{j,0}, & i=0;\\ p b_{m,0}, & 0<i<m, 0 \le k \le W_{i-1};\\ p(b_{m-1,k}+b_{m,0}), & i=m, 0 \le k \le W_{i-1}. \end{cases} \tag{8}$$

By imposing the normalization condition and considering (8), we can obtain $b_{0,0}$ as a function of p:

$$1 = \sum_{i=0}^{m} b_{i,0} \sum_{k=0}^{CW_1-1} \frac{CW_i - k}{CW_i(1-P_b)},$$
$$b_{0,0} = \frac{2(1-P_b)(1-2p)(1-p)}{(W_{\min}+1)(1-2p) + W_{\min} p\,(1-(2p)^m)}. \quad (9)$$

Therefore, we get the access probability of the SU as follows:

$$\tau = \sum_{i=1}^{M-1} b_{i,j} = \frac{b_{0,0}}{1-p} = \frac{2(1-P_b)}{W_{\min}+1+W_{\min} p \sum_{k=0}^{m-1}(2p)^k}, \quad (10)$$

where p is the collision probability of a packet in a given channel and is given as follows:

$$p = \left(1-(1-\tau)^{N_s^{1\mathrm{ch}-1}}\right) + (1-\tau)^{N_s^{1\mathrm{ch}-1}} P_b. \quad (11)$$

The throughput of the given channel is used to calculate the total system throughput over all channels considering the probability P_p of a channel being busy due to PUs' transmission. The probability of successful transmission, idleness, and collision, denoted by P^s, P^{id}, and P^c, respectively, are given by

$$P^s = N_s^{1\mathrm{ch}-1}\tau(1-\tau)^{N_s^{1\mathrm{ch}-1}}(1-P_b),$$
$$P^{\mathrm{id}} = N_s^{1\mathrm{ch}}\tau(1-\tau)^{N_s^{1\mathrm{ch}-1}}, \quad (12)$$
$$P^c = 1 - P^s - P^{\mathrm{id}}.$$

Assuming the probability that a channel can be used by SUs is $(1-P_p)$, the throughput S per channel is finally given by

$$S = \frac{P_s L(1-P_p)}{P^s T^s + P^c T^c + P^{\mathrm{id}} T^{\mathrm{id}}}, \quad (13)$$

where T^s is the average time needed to transmit a packet of size L (including the inter-frame spacing periods), T^{id} is the duration of the idle period (a single slot) and T^c is the average time spent in the collision.

V. GAME THEORETIC MODEL

In this section, we introduce some definitions and the terminologies from non cooperative game theory, which used in our paper. In many situations the theory of non cooperative games studies the behavior of selfish players where each player's optimal choice may depend on his forecast of the choice of his opponent. The word "non cooperative" means that the players' choices are based only on their perceived self interest and they do not try to find an agreement with other players [9].

In this section we analyze the behavior of misbehaving SUs in the presence of PUs using strategic (normal) form games. A game in strategic form has three elements: The set of players $i \in \mathbb{N}$, which we take to be finite set $\mathbb{N} = \{1, 2, \cdots, N\}$, the pure-strategy space S_i of each player i, and payoff functions u_i that give player i utility $u_i(s)$ for each profile $s = (s_1 \cdot s_I)$ of strategies. We denote all players other than player i by "$-i$" and their strategy profile by $s_{-i} = (s_1, \cdots, s_{i-1}, s_{i+1}, \cdots, s_N)$. In our model, the players are the SUs. The pure strategy of each player i is the contention window W_i and the utility function of each player is given by throughput r_i. We assume the cheaters in our model to be rational, i.e., they want to maximize their own benefit. In this particular context, the cheaters want to maximize the average throughput they receive by changing the contention window W_i. In our model there are N players out of which I are cheaters.

A. Variation of Throughput with W_i

The throughput achieved by a given node i, which is the average information payload transmitted in a time slot over the average length of a time slot can be computed as follows:

$$r_i^c = \frac{P_s^i L}{P^s T^s + P^c T^c + P^{\mathrm{id}} T^{\mathrm{id}}}, \quad (14)$$

where $P_i^s = \tau_i^{(c)} \Pi_{j\neq i}(1-\tau_j^{(c)})(1-\tau^{(w)})^{(N-I)}(1-P_B)$ is the probability that node i successfully transmits during a random time slot. L is the average packet payload size; $P^s = \Sigma_{i\in I}\tau_i^{(c)}\Pi_{j\neq i}(1-\tau_j^{(c)})(1-\tau^{(w)})^{(N-I)}(1-P_B)$; T^s is the average time needed to transmit a packet of size L (including the inter-frame spacing periods); $P^{\mathrm{id}} = \Pi_{i\in I}(1-\tau_i^{(c)})(1-\tau^{(w)})^{(N-I)}(1-P_B)$ is the probability of the channel being idle; T^{id} is the duration of the idle period (a single slot); $P^c = 1-P^s-P^{\mathrm{id}}$ is the probability of collision; and T^c is the average time spent in the collision. Note that $P^s+P^c+P^{\mathrm{id}}=1$ has to be satisfied. Since cheater i does not respect the backoff procedure of IEEE 802.11 (i.e., $m=0$ in equation (10), its channel access probability in the presence of PUs is given by

$$\tau_i^{(c)} = \frac{2(1-P_b)}{W_i+1}, \quad (15)$$

where W_i is the cheater i's contention window size. The channel access probability for well-behaved nodes, $\tau_j^{(w)}$, is

$$\tau_j^{(w)} = \frac{2}{W_{\min}+1+p^{(w)}W_{\min}\Sigma_{k=0}^{m-1}(2p^{(w)})^k}, \quad (16)$$

where

$$p^{(w)} = \left(1-(1-\tau^{(w)})^{N-I-1}\Pi_{i\in I}(1-\tau_i^{(c)})\right) + \left((1-\tau^{(w)})^{N-I-1}\Pi_{i\in I}(1-\tau_i^{(c)})\right)P_b. \quad (17)$$

Note that $\tau_j^{(w)}$ is the same for all the well-behaved nodes; so we set $\tau_j^{(w)} = \tau^{(w)}$. After arithmetic manipulation of the throughput (14) we obtain the following expression for throughput $r_{(i)}^c$ for cheater i:

$$r_{(i)}^c = \frac{\tau_i^{(c)} c_i^{(1)}}{\tau_i^{(c)} c_i^{(2)} + c_i^{(3)}} \quad (18)$$

where

$$c_i^{(1)} = p_{-i}L,$$
$$c_i^{(2)} = p_{-i}(T^s - T^{\mathrm{id}}) - s_{-i}(T^s - T^c),$$
$$c_i^{(3)} = (1-s_{-i}-p_{-i})T^c + s_{-i}T^s + p_{-i}T^{\mathrm{id}},$$

where $p_{(-i)}$ and $s_{(-i)}$ are substituted as follows:

$$p_{(-i)} = \prod_{j\in I\backslash\{i\}} (1-\tau_j^{(c)})(1-\tau^{(w)})^{(N-I)}(1-P_B),$$
$$s_{(-i)} = \sum_{j\in I\backslash\{i\}} \tau_j^{(c)} \prod_{k\in I\backslash\{i,j\}} (1-\tau_k^{(c)})(1-\tau^{(w)})^{(N-I)}(1-P_B).$$

Therefore the expected throughput of each node is a strictly decreasing function in terms of W_i for a specific value of P_b, the blocking probability of PUs. From (18), each node can obtain various throughputs by varying its contention window W_i. This can be shown as follows by assuming W_i for a constant P_b. By taking the first derivative in (18), we obtain the following decreasing function:

$$\frac{\partial r_i^c}{\partial W_i} = \frac{\partial r_i^c}{\partial \tau_i^{(c)}}\frac{\partial \tau_i^{(c)}}{\partial W_i} = \frac{c_i^{(1)}c_i^{(3)}}{(\tau_i^{(c)}c_i^{(2)}+c_i^{(3)})^2}. \tag{19}$$

The above equation is verified by our simulation using MATLAB. A network consists of $N = 20$ nodes randomly spread over an 100 m $\times$ 100 m area. We assume that all the nodes are within a receive range of each other. We use the parameters for the IEEE 802.11 protocol that are chosen according to the IEEE 802.11b standard [10] as given in Table 2. It is also assumed that no RTS/CTS handshake is used. Fig. 3 plots the throughput obtained by a random cheater i, as well as by each well-behaved node for different values of W_i and different value of P_b. Fig. 3 shows that a cheater can increase his expected payoff (received throughput) at the expense of other SUs by choosing a small value of W_i. But its payoff decreases when the channels are busy due to PUs. Therefore the cheaters can only increase their expected payoff at the expense of other well-behaved SUs' throughput but not at the expense of PUs' throughput.

B. Nash Equilibrium of the Game

In normal-form game, Nash equilibrium is a profile of strategies such that each player's strategy is an optimal response to the other players' strategies [9].

Definition 1 (Nash equilibrium) A strategy profile $W = (W_1, \cdots, W_I)$, which is the set of contention window values used by players, is a Nash equilibrium if and only if, for every player $i = 1, \cdots, I$,

$$r_i(W_i, W_{-i}) \geq r_i(W_i', W_{-i}). \tag{20}$$

According to (9), Nash equilibrium of our game is $W = 1$. A node gains the highest throughput if its access probability is equal to $1 - P_b$ which means $W = 1$. If only one node chooses $W = 1$ and the other nodes choose $W > 1$, the node with $W = 1$ gains positive throughput while the throughput of other nodes are zero. On the other hand, if more than one node choose $W = 1$, the throughput of all nodes will be zero. However, since we assume that each cheater tries to attain as high throughput as he can, it is most likely that more than one cheater sets $W_i = 1$. Therefore, this equilibrium is not efficient. The Nash

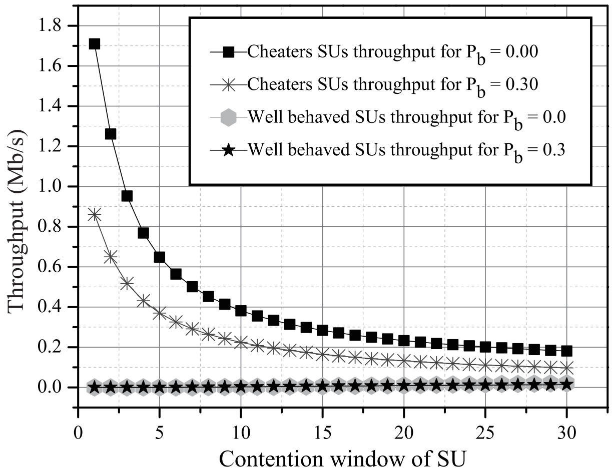

Fig. 3. Throughput for 20 nodes (out of which one is a cheater) in terms of various values of P_b.

equilibrium in this game is different from Nash equilibrium in the static game of [6], because it considers the presence of PUs. This equilibrium is shown in Fig. 3. The throughput of the cheater is maximum when $W_i = 1$. His throughput decreases when PUs occupy the channels.

C. Unique, Fairness, and Pareto-Optimal Point of the Game

In general a desirable solution for a game should exhibit the following three properties: (i) Uniqueness—this is to avoid uncertainties with respect to what solution each player should choose; (ii) fairness—the solution should result in a fair distribution of system throughput without interfering with PUs' throughput; (iii) Pareto optimality—the solution should result in a Pareto-optimal allocation of the throughput without interfering the PUs [10]. Based on the above analysis there are two types of Nash equilibrium points. One of the points is found when we have only a single cheater which gets all the payoffs and the rest of the SUs get zero payoffs. This results in unfair distribution of the throughput. The second equilibrium is found when every cheater simultaneously tries to access the channel all the time, by making its $W_i = 1$ and then causing repeated collisions. This is clearly inefficient and leads us to a question whether anything better can be achieved or not.

In order to derive a solution which is unique, fair and optimal, we use the Nash bargaining framework (NBF) [11]. An N-player Nash bargaining game consists of a pair (U, c), where $U \subseteq R_+,^N$ is a compact convex set and $u \subset U$. Set U is a feasible set and its elements give utilities that the N players can simultaneously accrue. Point u is a disagreement point that gives the utilities that the N players obtain if they decide not to cooperate. The set of N agents will be denoted by B and the agents will be numbered $1, 2, \cdots, N$. Game (U, u) is said to be feasible if there exists a point $v \in U$ such that $\forall i \in I, r_i > u_i$, and infeasible otherwise.

The solution to a feasible game is the point $v \in N$ that satisfies the following four axioms.

1. **Pareto optimality**: No point in U can weakly dominate v.

2. **Invariance under affine transformations of utilities**: If the utilities of any player are redefined by multiplying by a

scalar and adding a constant, then the solution to the transformed game is obtained by applying these operations to the particular coordinate of r.

3. **Symmetry**: If the players are renumbered, then it suffices to renumber the coordinates of r accordingly.

4. **Independence of irrelevant alternatives**: If r is the solution for (U, u), and $Q \subseteq R_+^n$ is a compact, convex set satisfying $u \in Q$ and $v \in Q \subseteq U$, then r is also the solution for (Q, u).

Definition 2 If a game (U, u) is feasible then there exists a unique point in N satisfying the axioms stated above. This is also a unique point that maximizes $\prod_{i \in U}(r_i - u_i)$, over $r \in U$.

Our game in cooperative game considers the secondary users which are cheaters in the presence of primary users. In this game the set of the joint feasible payoffs is given as follows:

$$U = \{r^c = (r_1^c, \cdots, r_I^c) : r_i^c = f_i(W), i \in I, W \in S\}, \quad (21)$$

where the functions $f_i(.)$ are derived from (14) and (16). The disagreement point of the Nash bargaining frame work is a fixed disagreement vector $u = (u_1, \cdots, u_I)$. For our model it is reasonable to define for every player $i \in I$ as follows:

$$u = \min_{W_{-i} \in S_{-i}} \max_{W_i \in S_i} r_i^c(W_i, W_{-i}) = 0.$$

Therefore, the disagreement point becomes

$$u_i = 0.$$

This implies that the corresponding strategy profile W is such that at least two or more players follow the strategy $W_i = 1$.

The above axioms are the sufficient condition for the bargaining problem B to be a unique solution. In addition to these axioms the payoffs have to be convex and compact (and there exists at least one feasible point strictly preferable to the disagreement) [12]. However, the set of joint payoffs N in the case of the game is neither compact nor convex: It consists of a countable finite number of points r_i^c. The maximization problem for the bargaining problem (U, u) is given as follows:

$$\begin{aligned} &\text{maximize } \Pi_{i \in I}(r_i^c - u_i) \\ &\text{subject to } r_i^c \in N \\ &\quad r^c \geq u. \end{aligned} \quad (22)$$

In order to make the problem convex function we take the logarithm of the objective function of (22) and using the fact $u_i = 0, \forall i \in I$ we obtain the equivalent maximization problem [12]:

$$\begin{aligned} P_1 = &\text{maximize } \sum_{i \in I} \log\{r_i^c(W_i)\} \\ &\text{subject to } r_i^c = f_i(W) \\ &\quad r^c \geq u \\ &\quad W \in S. \end{aligned} \quad (23)$$

Using the approach in [12] to solve the optimization problem we solve (23). Let us define a set $Y = \{Y_k \colon Y_k = \sum_{i \in I} f_i(W_i), W \in S_1 \times S_2 \times \cdots \times S_I, K = 1, 2, \cdots, w_{\max}^I\}$. Note that some $Y_k \in Y$ will have the same value (e.g., for $I = 4$, vectors $W = \{2, 5, 4, 3\}$ and $W' = \{5, 3, 4, 2\}$) and they are equivalent by using the operator $Y_k = \sum_{i \in I} f_i(W_i)$. By relaxing the constraint on P_1 and making W continuous we have the following problem.

$$\begin{aligned} P_2 = &\text{maximize } \sum_{i \in I} \log\{r_i^c(W_i)\} \\ &\text{subject to } \sum_{i \in I} r_i^c \in Y \\ &\quad r^c \leq \mathbf{M} \\ &\quad r^c \geq u, \end{aligned} \quad (24)$$

where r_i^c and $\forall i \in I$ are continuous variables, and **M** is a maximum throughput that can be reached by the cheaters. Since P_2 is a relaxed version of P_1, this yields $P_2 \geq P_1$. Therefore solving (24) means indirectly solving (24). One way of dealing with P_2 is to solve one instance $(Y_k \in Y)$ of the problem and then simply pick the instance that maximizes each $Y_k \in Y$ and the corresponding objective function [6]. To solve P_2 we define the corresponding objective function but before that we simplify the constraint as $\sum_{i=1}^I r_i^c = Y_k$. The Lagrangian dual of the P_2 is defined as [12]

$$\begin{aligned} L(r, \lambda, \alpha, \beta) = &\Sigma_{i=1}^I \log\{r_i^c(W_i)\} - \lambda(\Sigma_{i=1}^I r_i^c - Y_k) \\ &- \Sigma_{i=1}^I \alpha_i(u_i - r_i^c) - \Sigma_{i=1}^I \beta_i(r_i^c - M). \end{aligned} \quad (25)$$

Using the Karush-Kuhn-Tucker first order necessary conditions [12], we get

$$\begin{aligned} \frac{\partial L}{r_i^c} = \frac{1}{r_i^c} - \lambda + \alpha_i - \beta_i = 0, i = 1, \cdots, I \\ \lambda(\Sigma_{i=1}^I r_i^c - Y_k) = 0, \lambda \geq 0 \\ \alpha_i(u_i - r_i^c) = 0, \alpha_i \geq 0 \\ \beta_i(r_i^c - M) = 0, \beta_i \geq 0. \end{aligned} \quad (26)$$

We assume there exists a feasible vector W such that the optimal value of P_2, i.e., $\prod_{i \in I}(r_i^c u_i)$ is strictly positive; then $\alpha_i = \beta_i, i = 1, 2, \cdots, I$. If one cheater makes his contention window $W_i = 1$ then the throughput $r_i^c = \mathbf{M}$ but the throughput of the rest of the cheaters will be zero, i.e., $r_j^c = u = 0, \forall j \neq i$, which implies $\prod_{i \in I}(r_i^c - u_i) = 0$. Using this explanation, (24) is reduced to the following form:

$$\frac{1}{r_i^c} - \lambda = 0. \quad (27)$$

Replacing (25) into the first constraint of (24), i.e., $\sum_{i \in I} r_i^c = Y_k$, we finally get:

$$r_i^c = \frac{Y_k}{I}. \quad (28)$$

From (26) we found out that there is a unique solution to the bargaining problem P_1 and is fairly distributed among the SUs. In this case each SU gets equal throughput; i.e., every cheater has to send his packets with the same contention window without interfering PUs' communication. Unlike the solution to (19) which is Nash equilibrium, the solution in (26) is Pareto-optimal but not Nash equilibrium.

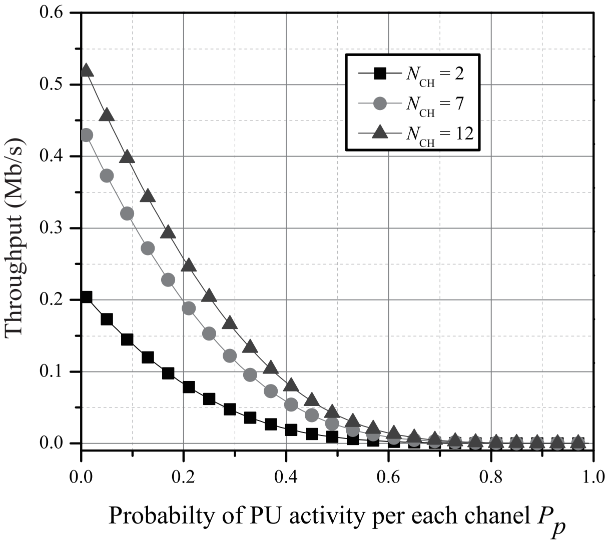

Fig. 4. Throughput of SUs with varying activity of PUs.

D. Penalizing Mechanism: Towards a Unique and Pareto-Optimal Nash Equilibrium

In the above section we have determined the desired point of operation and we now show one way of converging to the point without interfering with PUs. The main idea of this strategy is a penalty mechanism by which players can penalize severe deviations of another player [13]. Let us consider two arbitrary SUs, i and j from the set I. Assume that player i calculates the penalty p_j to be inflicted on player j as follows:

$$p_j = \begin{cases} r_j^c - r_i^c, & \text{if } r_j^c > r_i^c; \\ 0, & \text{otherwise.} \end{cases}$$

Therefore the throughput of player j is $r_j^c - p_j = r_i^c$ and the two SUs have the same throughput. The penalty mechanism used is jamming which is explained in details in [6], [13]. In this scheme if a player $i \in I$ detects the presence of non cooperative players other than PUs; first it calculates the throughput of deviating SUs and if it is higher than optimal throughput it changes to transmit mode and jams the player j according the (26).

VI. NUMERICAL RESULT

In this section, we first exhibit numerical results of the throughput performance of SUs based on (13). Fig. 4 shows the normalized throughput S per channel versus the number of orthogonal channels N_{CH} when the number of SUs is 20. The parameters used are listed in Table 2. According to our analysis, there is a Nash equilibrium point at which all the cheaters have $W_i = 1$, i.e., every cheater simultaneously tries to access the channel all the time, which result in repeated collisions. This is known as law of commons [9]. At this point all the cheaters get null throughput. This tells us how much the Nash equilibrium is inefficient when there are more than one cheaters. Let us consider an ideal scenario where $W_i = W$ and modify it to synchronize with other cheaters in the system. Now the set of the analysis consists of 20 nodes out of which there are 10 cheaters. The parameters are the same as listed in Table 2. Figs. 5–7 plot the average throughput obtained by a cheater at different values of W for different activity of PUs. All the figures show that if cheaters operate at $W = 1$ then it will result in network collapse or zero throughput for all cheaters. The throughput of all the cheaters also decreases when the PUs are using the channel. Fig. 5 shows that there exists an optimal point $(W = 27)$

Table 2. Parameters and values used for analysis.

Parameter name	Value
Packet payload	8,184 bits
MAC header	272 bits
PHY header	128 bits
ACK	112 bits + PHY header
Channel bit rate	2 Mbit/s
Propagation delay	1 μs
Slot time	50 μs
SIFS	28 μs
DIFS	128 μs

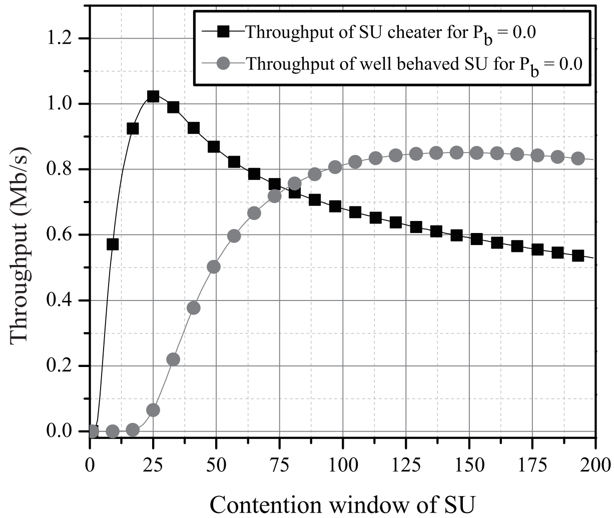

Fig. 5. Throughput vs. contention window size of cheaters (20 nodes, out of which 10 are cheaters) for $P_b = 0.0$.

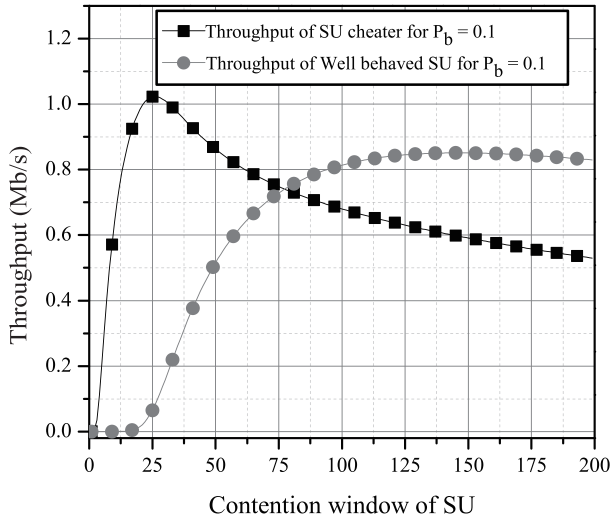

Fig. 6. Throughput vs. contention window size of cheaters (20 nodes, out of which 10 are cheaters) for $P_b = 0.1$.

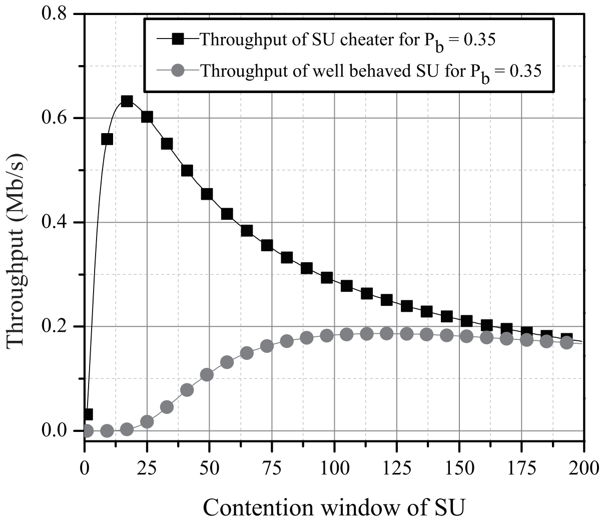

Fig. 7. Throughput vs. contention window size of cheaters (20 nodes, out of which 10 are cheaters) for $P_b = 0.35$.

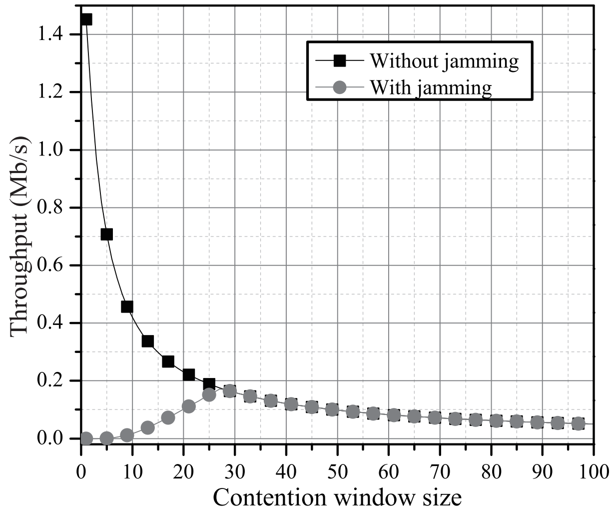

Fig. 9. Realizations of penalty through selective jamming for cheater X with or without the penalty mechanism for $P_b = 0.9$.

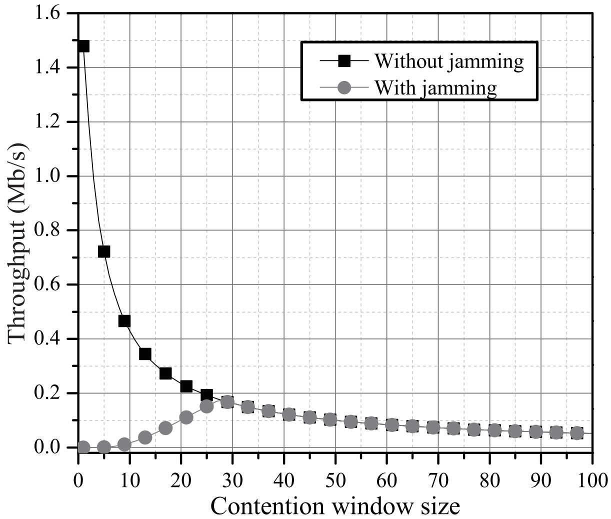

Fig. 8. Realizations of penalty through selective jamming for cheater X with or without the penalty mechanism for $P_b = 0.0$.

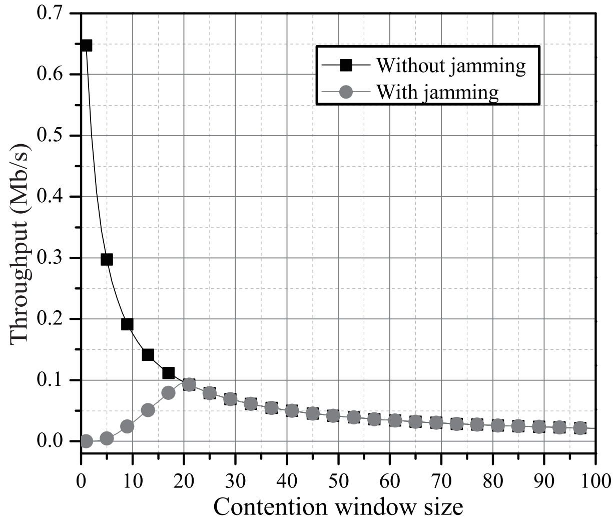

Fig. 10. Realizations of penalty through selective jamming for cheater X with or without the penalty mechanism for $P_b = 0.35$.

at which the throughput is maximized for every cheater in this system.

When PUs' activity increases in the system, this point remains the same but the throughput decreases as shown in Figs. 5–7. But it is not fair since the cheater gets the optimal point at the expense of the other well-behaved SUs. There is a point at which all the nodes intersect, which is known as Pareto-optimal [9]. A Pareto-optimal point means that it is impossible to move from that point in such a manner that the payoff enjoyed by other cheaters does not change. This is shown in (27) as a result of cooperation between the SUs. Moreover, the payoff of every cheater is maximized simultaneously. This point, however, varies with the presence of PUs, as shown in Figs. 5–7. As PUs' activity increases, this Pareto-optimal point moves towards more contention window and the throughput at this point also decreases.

Figs. 8–10 plot the average throughput obtained by cheater X when it unilaterally deviates from a given equilibrium point 29. As it is shown in the three figures when the user activity increases the overall throughput of the SUs decreases proportional with P_b. After the introduction of the detection, cheater X achieves the maximum throughput operating at the given equilibrium point where all cheaters set their contention window around to 29. This is predicted in (26), implying that this point is an equilibrium point where unilateral deviation is not profitable. Therefore this Nash equilibrium is fair and Pareto-optimal. By making an arbitrary contention window it is possible to create a Nash equilibrium point by using (27).

VII. CONCLUSION

In this paper we have analyzed the problem of cheating behavior of SUs in the presence of PUs. In order to accomplish the final result, first we have analyzed the throughput of SUs in the presence of PUs using a Markov chain model. Then we have used a game-theoretic approach to model the cheating behavior of some SUs. We have used cooperative game theory to find the optimal point. Jamming is assumed to punish the cheaters. In all the cases the analysis is done by considering the presence of the PUs which is the main contribution of this paper.

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